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CROSS STRESSES IN THE FLOW OF RAREFIED AIR

A. Foux
Research Associate

and

M. Reiner
Research Professor

Technian, Israel Institute of Technology, Haifa, Israel

ABSTRACT

The cross-stresses effect discovered by Reiner in the laminar flow of air was investigated experimentally for the case of rarefied air. It was found that the effect decreases with decreasing ambient pressure as long as the distance between the plates of the instrument is not reduced to the mean free path of the gas molecules, when the instrument becomes inoperative.

INTRODUCTION

The present paper reports on a continuation of a number of investigations in which was shown experimentally that a centripetal pumping effect, as previously found in high-polymer solutions, is present in gases. It was also shown that this effect is of second order in the velocity-gradient.

There are several possible explanations of the effect. One is based on the assumption that the fluid has the property of elastico-viscosity as first postulated by Maxwell, with a definite relaxation time. This can be accepted for a practically incompressible liquid. However, considering that in accordance with Maxwell the relaxation time of air is of the order 10^{-10} sec, one will hesitate to attribute to a gas the property of elastico-viscosity.

Sir Geoffrey Taylor explained the effect from an application of the Navier-Stokes equations, attributing it to geometrical and dynamical imperfections in the apparatus. However, his (and Dr. Saffman's) equations show that what may be called the Taylor-Saffman effect must increase with decreasing ambient pressure. In one of our previous investigations with slightly reduced ambient pressure, as well as in the present paper, it is shown experimentally that what Sir Geoffrey called the Reiner effect *decreases* with a decrease of the ambient pressure.

Another possible approach to the problem, in which the fluid is not considered as elastico-viscous, is by applying the constitutive equation of what Truesdell has named a Reiner-Rivlin fluid in which "cross-viscosity" makes its appearance. This approach has been followed by Srivastava (1961) and Rintel (1962). The result, however, is negative.

Maxwell's kinetic theory is in a certain sense primitive. Firstly, its model fits a monatomic gas only. Secondly, it considers first-order phenomena only. Chapman and Cowling (1952) and Ickenberry and Truesdell (1956) derived expressions for the second-order approximation. While following different methods, both arrived at the results from which it would appear that the second-order stress acts centri*fugally*—i.e., in opposition to the Reiner effect. Foux and Reiner (1962) showed experimentally that monatomic gases such as neon, argon, and helium in fact *do* show the centripetal effect. This points to an essential deficiency in the kinetic theory of gases.

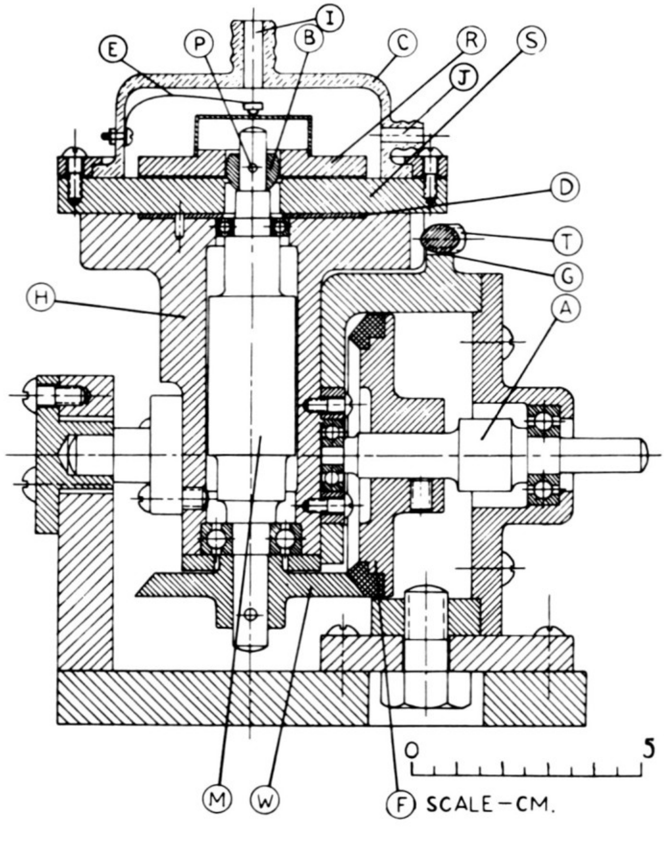


Fig. 1. Section of centripetal vacuum pump.

Though the Reiner effect, which is of second order, is of minute magnitude under ordinary conditions, it may be of primary importance in boundary cases, as for example, in extremely high rates of shear of the gas. In this case the calculated lift of a winged vehicle, based on the classical kinetic theory of gases, may vary by an essential factor.

DESCRIPTION OF APPARATUS

The apparatus used in the present investigation was the vacuum pump described in the papers by Popper and Reiner (1958) and Reiner (1958). It is shown in Fig. 1 in a section and illustrated in the photograph Fig. 2. S is the stator, R the rotor. The latter is driven by the main shaft M. However, R is not fixed to M; it can rotate freely around the ball B, and it can be displaced along the axis of rotation. It rotates together with the shaft about the axis of M. This is achieved by means of the pins P, which protrude from B, and enter into two short radial slots in R. During rotation, the cross-force in the direction of the velocity gradient, which is a pressure, maintains a gap (d) between the plates, thus supporting the rotor. The force (P), which tends to bring

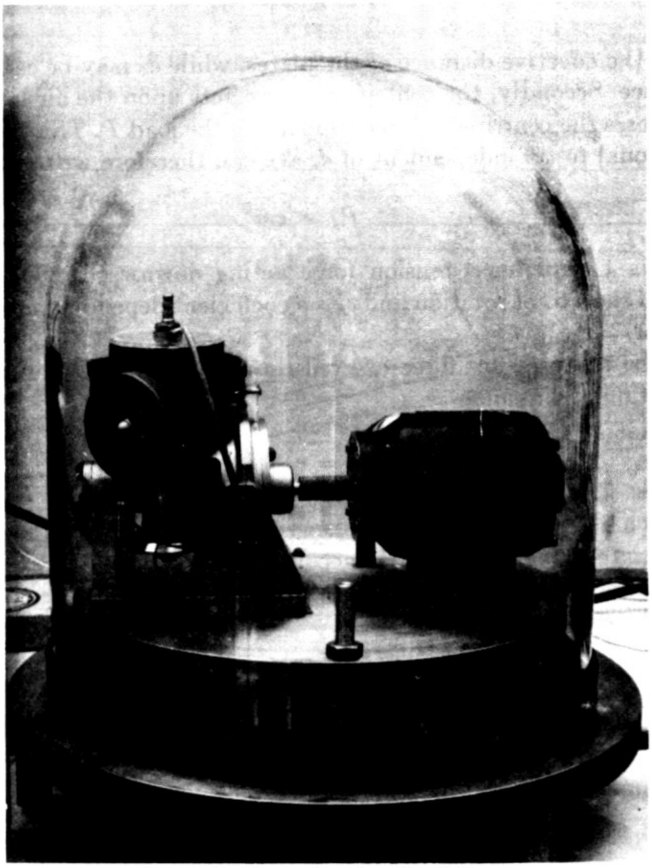


Fig. 2. Instrument under glass bell.

the plates together, is the sum of the weight (W) of the rotor and the compressive force (P_s) from the spring E. The rotor is insulated electrically from the stator. The plates thus form a capacitor by which the mean width (D) of the gap between the plates can be determined. The dielectric constant of all gases was taken as unity and edge effects were neglected.

In order to determine the effects in reduced pressures the instrument was put under a glass bell as shown in Fig. 2.

BASIC RELATIONS

In a series of experiments with air, which were reported previously by Reiner (1958), it was shown that the distance d increases with increasing square of velocity ω^2. So would d^2. The ratio $(d/\omega)^2$ may therefore be considered as a measure of the effect. However, it was found that $(d/\omega)^2$ is not a constant. This can be understood on two lines. Firstly, the gap distance D, as found from capacity measurements which is a mean distance, will not be identical with the thickness d of the laminar layer in which the flow takes place. In general,

$$D = d + d_0 \tag{1}$$

where d is the effective distance of the plates, while d_0 may be called the ineffective distance. Secondly, the centri*fugal* force acts upon the air layer in the gap, which opposes the centripetal force supporting the load P. This centrifugal force is proportional to ω^2, independent of d. We can therefore write

$$P_f = c_f\omega^2 \tag{2}$$

where P_f is a centrifugal tension force acting downward on the rotor in the direction of the axis of rotation and c_f is a coefficient depending upon the radius R of the rotor.

Let P_p be the pressure force exerted upward on the rotor through the centripetal action and assume

$$P_p = c_p\left(\frac{\omega}{d}\right)^2 \tag{3}$$

where c_p is a coefficient depending likewise upon R, and some factors for which an appropriate theory will have to account.

We can therefore try as an empirical relation based on the equilibrium condition of the rotor

$$P_p = P + P_f \tag{4}$$

or

$$P = c_p\left(\frac{\omega}{d}\right)^2 - c_f\omega^2 \tag{5}$$

from which

$$d^2 = \frac{c_p}{c_f + 1/\frac{\omega^2}{P}} \tag{6}$$

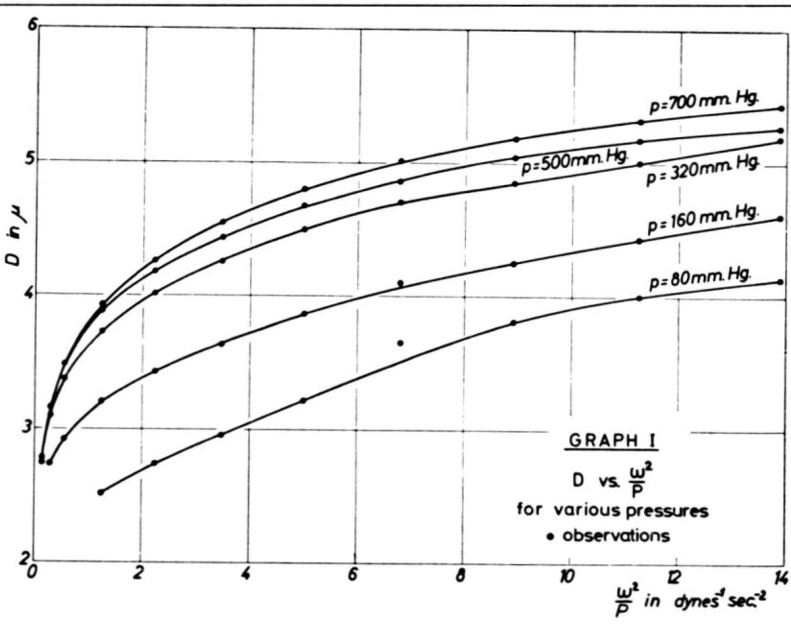
EXPERIMENTS IN REDUCED PRESSURE

Experiments in which the gap width was determined for different speeds of rotation at room temperature were carried out with nitrogen at ambient pressures of 700, 500, 320, 160, and 80 mm Hg. Two series of observations, each one consisting of four tests, were conducted at one pressure with speeds ranging between 1,000 to 10,000 rpm. The average results are listed in Table 1 and shown graphically in Figs. 3 and 4.

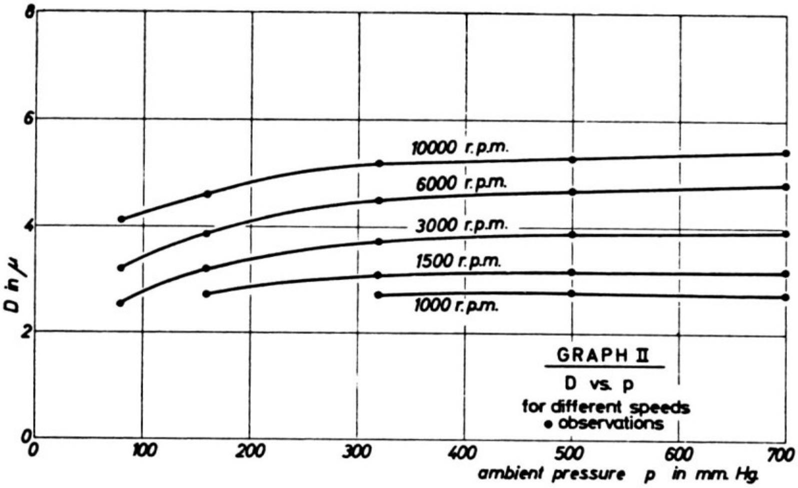
Table 1. *D* AS A FUNCTION OF SPEED OF ROTATION OF VARIOUS PRESSURES

		D in Microns for Ambient Pressure of				
Ω	ω^2/P	700 mm Hg	500 mm Hg	320 mm Hg	160 mm Hg	80 mm Hg
1	0.138	2.74	2.77	2.74	—	—
1.5	0.312	3.16	3.16	3.10	2.73	—
2	0.554	3.47	3.46	3.37	2.93	—
3	1.250	3.92	3.88	3.73	3.20	2.52
4	2.226	4.26	4.18	4.01	3.42	2.74
5	3.482	4.54	4.43	4.25	3.63	2.92
6	5.001	4.80	4.67	4.49	4.86	3.21
7	6.812	5.01	4.86	4.70	4.10	3.65
8	8.903	5.18	5.04	4.85	4.25	3.82
9	11.250	5.32	5.17	5.00	4.43	4.01
10	13.899	5.43	5.26	5.18	4.61	4.13

Ω in 10^3 rpm; ω^2/P in dynes^{-1} sec^{-2}



Graph I.



Graph II.

The average results recorded in Table 1 appear to be reliable. They fell within 5 percent of each individual test, at the extreme cases in the high velocity range and low pressure, where the instrument showed some stability. At lower speeds and higher pressures the variations were much less and fell within the 1 percent error.

At pressure lower than 80 mm Hg the instrument failed to operate. At this pressure the gap width was the same as the mean free path of the nitrogen molecules. For a lower axial force P, that is a lighter rotor, the width of the gap would increase, but it is difficult to obtain a highly balanced rotor of a smaller weight.

DETERMINATION OF PARAMETERS

The basic relation from Eqs. (1) and (6)

$$D = d_0 + \sqrt{c_p/\left(c_f + 1/\frac{\omega^2}{P}\right)} \tag{7}$$

was used to determine the constant d_0, and the parameters c_p and c_f. The values of D for three values of ω^2/P (the lowest, 2 and 8) were substituted in Eq. (7), thus producing three simultaneous equations with the three unknowns d_0, c_p, and c_f. For the results obtained with an ambient pressure of 80 mm Hg, the first three experimental observations were used.

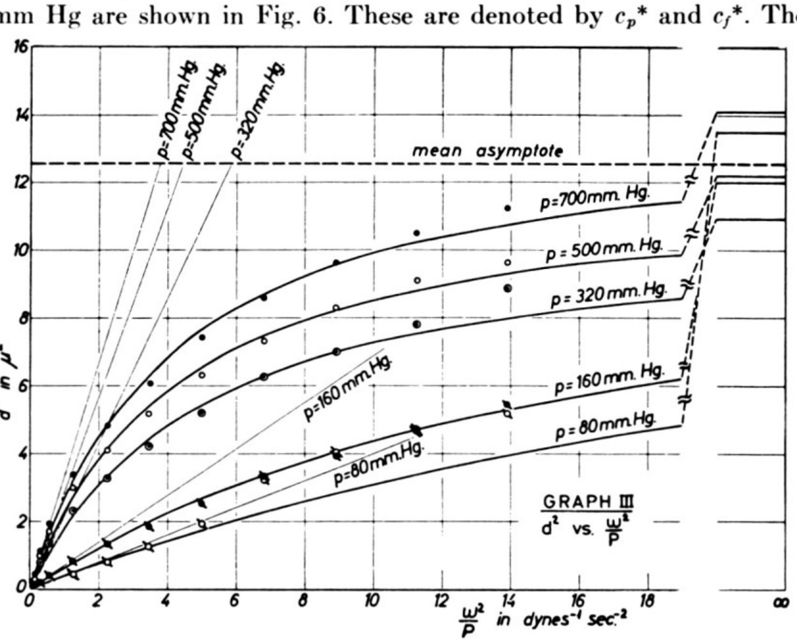
The constant d_0 and parameters c_p and c_f determined by this method are shown in Table 2, for the different pressures.

Table 2. CALCULATED PARAMETERS FOR VARIOUS PRESSURES

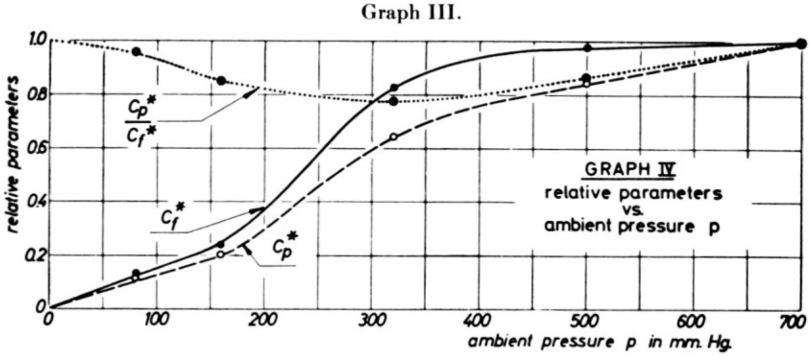
Pressure in mm Hg	d_0 in μ	C_p in 10^{-8} dynes sec² cm²	C_f in dynes sec²
700	2.07	3.33	0.24
500	2.16	2.82	0.23
320	2.20	2.15	0.20
160	2.27	0.68	0.06
80	1.82	0.40	0.03

A comparison of the theoretical curves based on Eq. (6) and the parameters c_p and c_f as found in Table 2, and the experimentally found distance D minus d_0, can be seen in Fig. 5.

The relative parameters c_p and c_f with respect to those of ambient pressure of 700 mm Hg are shown in Fig. 6. These are denoted by c_p^* and c_f^*. The ratio



Graph III.



Graph IV.

c_p^*/c_f^* is also shown. As can be seen from Eq. (6), there is an asymptotic approach of the theoretical curve for $\omega^2/P \to \infty$, to the line $d^2 = c_p/c_f$. The ratio c_p^*/c_f^* is therefore an indication of the relative position of the asymptote. The graph shows within observational errors a constant value for c_p^*/c_f^*.

CONCLUSIONS

It has been shown that when air (represented by N_2) is brought into torsional flow between two circular discs, at high rates of shear, a self-acting air thrust bearing is produced. This presupposes the appearance of a centripetal action, characterized by a factor c_p, in addition to the classical centrifugal action characterized by c_f. Let ω be the angular velocity of the rotating disc, and d the active distance between the discs, then the centrifugal action can be expressed by

$$P_f = c_f\omega^2$$

while the centripetal action is

$$P_p = c_p\left(\frac{\omega}{d}\right)^2$$

The thrust bearing force is

$$P = P_p - P_f$$

The effect vanishes when the gap width d becomes smaller than the mean free path of the gas molecules. In the instrument described, the force P is kept constant, and d is found as a function of ω^2/P in accordance with

$$d^2 = \frac{c_p}{c_f + 1/\frac{\omega^2}{P}}$$

with an asymptote at $d = c_p/c_f$. These actions diminish with diminishing ambient pressure, roughly in the same proportion, so that the ratio c_p/c_f is roughly constant. In a preceding investigation [Foux and Reiner (1962)] it was shown that this ratio is also of the same magnitude for different (including monatomic) gases. We therefore must conclude that this ratio indicates the existence of a universal gas constant for which the classical kinetic theory of gases does not account.

ACKNOWLEDGMENT

Thanks are due to Professor Schnepp, Head of the Department of Chemistry, for his help in providing some of the instruments. The following members of the staff of the Department of Mechanics participated in the execution of the experiments and preparation of the present paper: Messrs. P. Schechter, G. Stavi, and Mrs. G. Frankenfeld.

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DISCUSSION

Authors: A. Foux and M. Reiner
Discussor: Felix E. Nagel, Melpar, Inc.

I wish to congratulate the authors for this quite interesting paper. The basic assumption was made that the speed varies linearly between the adjacent plates, and the stress tensor is based on this very assumption. I presume that in the term $\delta_{ij}p$ for this tensor the δ_{ij} refers to the Kronecker Delta. We have a very special boundary condition of shear flow, which of course in the general case of a body moving through a fluid will not obtain. But even then a linear variation is an approximation resembling Poiseuille flow at low velocities.

Can the author give the limiting value of Reynold's number for his assumed flow? The velocity gradient at the moving boundary surface yields the skin friction if the viscosity of the gas is known, and could verify the assumed linear velocity distribution, which may only be present for highly rarefied gases.

(*Author made no reply.*)